Human Capital Investment under Quasi-Geometric Discounting

Karl David Boulware
Robert R. Reed*
Ejindu Ume

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Abstract

Recent work by Laibson (1997) identifies that individuals are excessively impatient in the short-run, but wish to become more patient over time. It is often argued that such a time-inconsistency problem distorts individuals' savings decisions. The objective of this paper is to study human capital accumulation in the presence of a time-inconsistency problem. In doing so, we explain that many policies put into place to take advantage of the inter-personal benefits from human capital accumulation may also be important for resolving 'intra-personal' planning problems. Our results also shed light on the role of compulsory education.

*Karl David Boulware, Department of Economics, Finance, and Legal Studies, University of Alabama, Tuscaloosa, AL 35487; Email: kdboulware@gmail.com; Robert R. Reed, Corresponding Author, Associate Professor, Department of Economics, Finance, and Legal Studies, University of Alabama, Tuscaloosa, AL 35487; Email: rreed@cba.ua.edu; Ejindu Ume, Department of Economics, Finance, and Legal Studies, University of Alabama, Tuscaloosa, AL 35487; Email: esume@crimson.ua.edu.

1 Introduction

Since the seminal work of Lucas (1988), the role of human capital as an engine of economic activity has gathered a great deal of attention. In particular, Lucas emphasizes that interactions among people are an important component of human capital accumulation: "human capital accumulation is a social activity involving groups of people." (p. 19) Moreover, it is often argued that human capital accumulation is inefficient due to the presence of external economies. As evidence of external economies from human capital, Rauch (1993) observes important productivity effects from human capital accumulation. Notably, he finds that individuals in metropolitan statistical areas with higher average years of education earn higher wages after controlling for population size. Because of their important social benefits, human capital is subsidized in a number of ways.

The objective of this paper is to argue that public policies directed towards human capital investment may be more important than previously acknowledged. The root of our position is based upon the inability of individuals to commit to future actions. As emphasized in recent work by Laibson (1997), individuals' rates of time preference evolve over time. That is, an individual's short-run rate of time preference differs from the long-run rate. As an example, individuals often express that they would like to save in the future, but do not intend to begin saving substantially more today. Embedded in this statement is the idea that individuals believe that they should be patient in the future, but are currently relatively impatient. As a result, an individual's accumulated savings is likely to be inefficiently low.

In contrast to previous work on human capital accumulation which studies the implications of externalities among people for economic activity, we study human capital investment in a setting where individuals face an 'intra-personal' externality. This takes place because individuals' choices are time-inconsistent and people play a strategic game against their future selves because the way they evaluate the future evolves over time. Since individuals cannot commit to future actions, standard optimization techniques cannot be used. Instead, Krusell, Kuruscu, and Smith (2002) demonstrate that the standard recursive tools used to study the neoclassical growth model can be adopted if individuals suffer from quasi-geometric discounting. Taking advantage of the tractability offered by adopting specific functional forms, Krusell et. al. study Markov perfect equilibria. The solution to the problem is time-consistent as none of the individual's future selves would choose to deviate from the savings plan. Since the solutions to the game are closed-form, the equilibrium to the game is unique.

Rather than studying an economy with physical investment, we examine human capital accumulation in the presence of quasi-geometric discounting. As in Krusell et. al., individuals have logarithmic preferences. However, there is not an explicit savings decision. That is, all output is consumed in each period. Similar to Lucas, the opportunity cost of investment in human capital is foregone income – human capital accumulation promotes knowledge and raises the productivity of labor effort in each period. However, in contrast to Lucas, we are not particularly interested in studying a growth economy. Instead, our objective is to examine the implications from the time-inconsistency problem arising from differences in the short-run and long-run rates of time preference in a general equilibrium framework. Consequently, we assume that human capital accumulation is limited by diminishing returns from time spent studying.

The remainder of the paper is as follows. Section 2 introduces the basic setup of the model and shows that recursive methods are necessary for studying human capital investment in the presence of quasi-geometric discounting. Section 3 follows by examining human capital investment through the methods produced by Krusell et. al. Section 4 offers some concluding remarks.

2 The Structure of the Model and the Need for Recursive Methods

We begin by outlining the structure of the model. We consider a consumer representative who is capable of dictating the amount of time to spend studying, but also inherits the consumer's preferences with quasi-geometric discounting. As in Krusell et. al., an individual's utility function is logarithmic:

$$\omega(c_t) = \ln(c_t)$$

The agent's preferences evolve over time as specified by the following quasi-geometric discount function:

$$\Omega_0(c) = \omega(c_0) + \delta[\beta\omega(c_1) + \beta^2\omega(c_2) + \beta^3\omega(c_3) + ...]$$

Notably, β represents the standard "geometric" component to discounting future utility. However, an individual's degree of short-run patience differs from the long-run rate of time preference. That is, $\delta\beta$ reflects the rate at which an individual discounts utility from the following period while β represents the rate at which utility is discounted any period thereafter. Consequently, $\delta\beta$ reflects an individual's short-run rate of time preference while β reflects the long-run rate. Obviously, if $\delta < 1$, the short-run rate differs from the long-run rate. In these circumstances, the way that the individual values the future evolves over time. As we elaborate below, individuals cannot commit to future human capital investment because once the future arrives they become more impatient than they did at an earlier point in time.

Output in each period is determined by labor effort in production and the individual's level of human capital:

$$y_t = Au_th_t$$

With the remaining amount of time, the individual will accumulate human capital:

$$h_{t+1} = B(1-u_t)^{\phi}h_t^{1-\phi}$$

As mentioned in the introduction, we are not focused on studying an economy with positive growth as in Lucas. Instead, our intention is merely to show how human capital accumulation is distorted by the time-inconsistency problem. Consequently, we impose that individuals experience diminishing returns from time spent studying in the production of new human capital.

As we will explain below, the Markov perfect equilibrium to the dynamic game is formulated as a game in which the agent plays a game against the agent's future selves. From this perspective, *an individual will be relatively impatient in the future and free-ride off of the human capital acquired by previous generations.* In Lucas, the parameter γ in the production of the consumption good represents the free-rider problem between agents at a given point in time. *However, in our setup, the agent has an intra-personal free-rider problem in the production of human capital. That is, the intra-personal externality depends on both δ and $(1-\phi)$.*

Since there is not a store of value in the economy, consumption in each period is based entirely on output:

$$c_t = Au_t h_t$$

The individual's choice of working time today has consequences for the future because of human capital accumulation:

$$u_t = 1 - \left(\frac{h_{t+1}}{Bh_t^{1-\phi}}\right)^{\frac{1}{\phi}}$$

We may then write the problem in terms of the amount of human capital to acquire for use in the following period:

$$\Omega_0(c) = \omega(h_0, h_1) + \delta\beta\omega(h_1, h_2) + \delta\beta^2\omega(h_2, h_3) + ...$$

In order to maximize lifetime utility as of period t, the individual's choice of human capital to acquire for period $t+1$ is given by:

$$\frac{\partial\omega(h_t, h_{t+1})}{\partial h_{t+1}} = \delta\beta\left(\frac{\partial\omega(h_{t+1}, h_{t+2})}{\partial h_{t+1}}\right)$$

By comparison, the choice for period $t+2$ is:

$$\frac{\partial\omega(h_{t+1}, h_{t+2})}{\partial h_{t+2}} = \beta\left(\frac{\partial\omega(h_{t+2}, h_{t+3})}{\partial h_{t+2}}\right)$$

Yet, once period $t+1$ arrives, the individual's trade-offs between the current and future become:

$$\frac{\partial\omega(h_{t+1}, h_{t+2})}{\partial h_{t+2}} = \delta\beta\left(\frac{\partial\omega(h_{t+2}, h_{t+3})}{\partial h_{t+2}}\right)$$

Thus, the individual at time $t+1$ would disregard the human capital investment plan put into place at time t. As a result, individuals with quasi-geometric discounting cannot commit to future plans. In order to find the solution to this problem, recursive methods must be applied.

3 The Time-Consistent Solution

To arrive to the time-consistent solution, Krussel, Kuruscu, and Smith (2002) formulate the agent's problem as a recursive game of the current agent against the agent's future selves. In fact, Krussel et. al. study Markov perfect equilibria. As a result of the Markov assumption, the policy function for savings only depends on current savings. Krussel et. al. examine the level of capital accumulation in such a game in a setting with functional forms so that the model yields closed-form solutions. The solution is time-consistent because future selves would choose not to deviate.

In comparison to Krussel et. al, we study human capital accumulation. In our framework, current income is endogenous and depends on the amount of time invested in human capital. Moreover, it also depends on the amount of human capital investment by the individual's prior selves. Instead, current income in the neoclassical growth model is pre-determined.

To begin the analysis of the recursive method, let the current self's perception of future human capital investment be determined by a function $h_{t+1} = g(h_t)$. The current consumer-representative solves the following problem:

$$V_0(h) = \overset{Max}{h'} \left[\log \left(\frac{B^{\frac{1}{\phi}} h^{\frac{1-\phi}{\phi}} - (h')^{\frac{1}{\theta}}}{B^{\frac{1}{\phi}} h^{\frac{1-\phi}{\phi}}} \right) + \log h_t + \delta \beta V(h') \right] \tag{1}$$

where

$$V(h) = \overset{Max}{h'} \left[\log \left(\frac{B^{\frac{1}{\phi}} h^{\frac{1-\phi}{\phi}} - (g(h))^{\frac{1}{\phi}}}{B^{\frac{1}{\phi}} h^{\frac{1-\phi}{\phi}}} \right) + \log h_t + \beta V(g(h)) \right] \tag{2}$$

The current solution for the problem (as of time 0) is $\widetilde{g}(h)$. A time-consistent solution occurs when $\widetilde{g}(h) = g(h)$ for all h.

The Markov perfect solution to the problem under a no-commitment regime is defined as:

Definition. The first-order Markov perfect solution to the consumer-representative's problem consists of a decision rule, $g(h)$, a value function, $V(h)$, and a law of motion for the stock of human capital, $h' = g(h)$, such that:

1. Given $V(h)$, $g(h)$ solves the maximization problem above;

2. Given $g(h), V(h)$ solves the functional equation above.

The solution to the problem is summarized in the following proposition:

Proposition. The first-order Markov perfect solution to the above problem under a no-commitment regime consists of:

1.

$$V(h) = a + b\log(h), \text{ where } b = \frac{(1-\phi)}{\phi\left[1-(1-\phi)\beta\right]}$$

2.

$$g(h) = \left(\frac{\delta\beta\,(1-\phi)}{1-(1-\delta)\beta(1-\phi)}\right)^{\phi} Bh^{1-\phi}$$

Lemma 1. The steady-state stock of human capital is:

$$h = \left(\frac{\delta\beta\,(1-\phi)}{1-(1-\delta)\beta(1-\phi)}\right) B^{\frac{1}{\phi}}$$

Let $\beta_0 = \delta\beta$ and $\beta_1 = \beta$. Thus, β_0 is defined as the short-run rate of time preference and β_1 as the long-run rate. If $\left(\frac{1-\beta(1-\phi)}{\beta(1-\phi)}\right)^2 > \delta$, then $\frac{\partial h}{\partial \beta_0} > \frac{\partial h}{\partial \beta_1}$.

By Lemma 1, a reduction in the time-inconsistency problem affects human capital accumulation more than an increase in the long-run rate of time preference if the time-inconsistency problem is sufficiently severe. *In other words, the distortion from the time-inconsistency problem can have a significant impact on human capital investment.*

It is commonly viewed that subsidies to human capital investment can have important welfare effects because of the social benefits from human capital – the benefits from inter-personal externalities. Interestingly, our results imply that they may also be important for alleviating the costs of intra-personal externalities due to the inability to commit to future actions.

It seems that society is aware of the problem of excessive short-run impatience, especially at an early age. For example, education is compulsory in all advanced countries through the age of 14. Many states in the U.S. require school attendance until the age of 18 – psychologists have documented that much of the development of the frontal lobe of the brain occurs during adolescence; the frontal lobe is primarily the part of the brain in which advanced functions such as planning and impulse control occur. Compulsory education is likely to be an important reaction to this problem at a young age.

References

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4 Appendix

Solve for the amount of time spent working:

$$u_t = 1 - \left(\frac{h_{t+1}}{Bh_t^{1-\phi}}\right)^{\frac{1}{\phi}}$$

The time-inconsistent value function is given by:

$$V(h_t) = Max_{h_{t+1}}\left[\log\left(B^{\frac{1}{\phi}}h_t^{\frac{1-\phi}{\phi}} - h_{t+1}^{\frac{1}{\phi}}\right) + \delta\beta b\log(h_{t+1})\right]$$

Given the guess of the value function, solve for h_{t+1} as a function of b:

$$h_{t+1} = \left(\frac{\phi\delta\beta b}{1+\phi\delta\beta b}\right)^{\phi} Bh_t^{1-\phi} \tag{3}$$

With the choice of human capital investment from (**??**), substitute back into the time-consistent Bellman equation in (**??**) in order to solve for b:

$$b = \frac{(1-\phi)}{\phi\left[1-(1-\phi)\beta\right]}$$

Use b to solve for h_{t+1} :

$$h_{t+1} = \left(\frac{\phi\delta\beta\left[\frac{(1-\phi)}{\phi[1-(1-\phi)\beta]}\right]}{1+\phi\delta\beta\left[\frac{(1-\phi)}{\phi[1-(1-\phi)\beta]}\right]}\right)^{\phi} Bh_t^{1-\phi}$$